Schur multipliers and combinatorics

Abstract

An infinite matrix *A* with complex entries is a Schur multiplier if, whenever *X* is the matrix of a bounded linear operator on Hilbert space, then so is the entrywise product *A* * *X*. While Schur multipliers have a beautiful functional-analytic characterisation due to Grothendieck, this can be difficult to apply in practice and it is often difficult to determine whether a matrix is a Schur multiplier or not.

A Schur multiplier *A* is idempotent if it is a matrix of 0s and 1s. We can then identify *A* with the bipartite graph containing the edges (*i*,*j*) for which the (*i*,*j*)-entry of *A* is 1. This combinatorial language is used in [2] to show that there are several "gaps" in the set of norms of idempotent Schur multipliers, extending results of Livshits [3] and Katavolos and Paulsen [1].

In this research project, we will continue to use both analytic and combinatorial tools to study Schur multipliers. Challenging motivating questions include:

- Can we characterise the set of graphs of idempotent Schur multipliers?
- Can we describe the set of norms of all idempotent Schur multipliers?

We will also consider variants of these questions in which we replace the set of bounded operators on a Hilbert space with another von Neumann algebra in the spirit of [4], or with another operator space.

A student hoping to work in this area should have a good grasp of basic functional analysis and an enthusiasm for combinatorics.

Bibliography

[1] Aristedes Katavolos and Vern Paulsen, *On the ranges of bimodule projections*, Canad. Math. Bull. **48** (2005), 91-111.

[2] Rupert Levene, *Norms of idempotent Schur multipliers*, New York J. Math. **20** (2014), 325-352.

[3] Leo Livshits, *A note on 0-1 Schur multipliers*, Lin. Alg. Appl. **22** (1995), 15-22.

[4] Florian Pop and Roger Smith, *Schur products and completely bounded maps on the hyperfinite II_1 factor*, J. London Math. Soc. (2) **52** (1995), 594-604.